Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year (تخلفات)	Final Term Exam Date: May 17, 2015 Course: Mathematics 1 – B Duration: **3** hours

• The Exam consists of one page Answer **All** Questions • No. of questions: 4 Total Mark: 100

Question 1

(a)If $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix}$. Show that A\`.A is symmetric matrix and find |A\`.A|. 5

(b)If $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$. (i)Find the eigenvalues and eigenvectors of A. 6

(ii)Write Hamilton equation and find A^{-1}. 5

(iii)Find the eigenvalues of $f(A) = \sqrt{A^2 + A}$ and $f(A) = 2^A$ 4

(c) If $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$. Find, if possible, A + B, A.B, A.B\`. 5

Question 2

(a)Using the binomial theorem, expand : $\frac{1}{\sqrt{x+9}}$ 4

(b) Solve the linear system: 3x + y – z = 2, 2x – y = 1, x + 2y – z = 6. 5

(c) Find S , S_{20} from each series: (i) $\sum_{r=1}^{n}(r+1)(2+r^2)$ (ii) $\sum_{r=1}^{n}\frac{3}{r^2+r}$ 8

(d)Using the mathematical induction, prove that:

(i) $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$ (ii) $n^3 + 2n$ is divisible by 3 8

Question 3

(a)Transform the equation $x^2 - 6xy + 9y^2 + 4x + 8y + 15 = 0$ to new axes through (– 2, 1). 7

(b)A point moves so that its distance from the x – axis is half of its distance from the point (2, 3). Find the equation of its locus. 6

(c)Find the value of λ such that the equation $2x^2 - 3xy - 2y^2 - 8x + 6y + \lambda = 0$ represents a pair of lines then fined the angle between them. 6

(d)Find the equation of the ellipse whose foci (±4,0) and its eccentricity is 1/3. 6

Question 4

(a)Find the equation of the tangent to the circle $x^2 + y^2 = 13$ at the point (2, 3). 6

(b)Find the equation of the parabola whose Focus is (1,2) and directrix is x + y – 2 = 0. 6

(c)Find the eccentricity, foci coordinates, directrix for the ellipse $4x^2 + 9y^2 = 144$. 6

(d)Find the equation of the tangent to hyperbola $4x^2 - 9y^2 = 36$ which parallels to the line y = 2x + 3. 7

Good Luck *Dr. Mohamed Eid* *Dr. Fathi Abdsallam*

Answer

Question 1

(a) $A^{`}.A = \begin{bmatrix}1 & 2\\3 & 0\\1 & 2\end{bmatrix}\begin{bmatrix}1 & 3 & 1\\2 & 0 & 2\end{bmatrix} = \begin{bmatrix}5 & 3 & 5\\3 & 9 & 3\\5 & 3 & 5\end{bmatrix}$. We see that A`.A is symmetric.

-- ***(5 Marks)***

(b)(i) $|A - \lambda I| = \begin{vmatrix}1-\lambda & 2\\3 & 2-\lambda\end{vmatrix} = (1-\lambda)(2-\lambda) - 6 = \lambda^2 - 3\lambda - 4 = 0$

Then, the eigenvalues are: $\lambda_1 = 4, \quad \lambda_2 = -1.$

From the equation, $\begin{bmatrix}1-\lambda & 2\\3 & 2-\lambda\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = 0$

For : $\lambda_1 = 3$, $\begin{bmatrix}-3 & 2\\3 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = 0$

Then – 3x + 2y = 0, 3x – 2y = 0

Then 3x = 2y = any number except 0

Put y = 3, we get x = 2 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix}2\\3\end{bmatrix}$

For : $\lambda_2 = -1$, $\begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = 0$

Then 2x + 2y = 0, 2x + 2y = 0

Then y = – x = any number except 0

Put x = 1, we get y = – 1 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix}1\\-1\end{bmatrix}$

-- ***(6 Marks)***

(b)(ii)The Hamilton equation is: $A^2 - 3A - 4I = 0$

Since |A| = – 4. Then inverse A exists. Multiply the Hamilton equations by A^{-1}.

Then $A - 3I - 4A^{-1} = 0$.

Then $A^{-1} = \frac{1}{4}(A - 3I) = \frac{1}{4}\left(\begin{bmatrix}1 & 2\\3 & 2\end{bmatrix} - \begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}\right) = \frac{1}{4}\begin{bmatrix}-2 & 2\\3 & -1\end{bmatrix}$

-- ***(5 Marks)***

(b)(iii)The eigenvalues of $f(A) = \sqrt{A^2 + A}$ are $f(4) = \sqrt{20}$, $f(-1) = \sqrt{0}$.

The eigenvalues of $f(A) = 2^A$ are $f(4) = 2^4 = 16$, $f(-1) = 2^{-1} = \frac{1}{2}$.

-- ***(4 Marks)***

(c) $A + B = \begin{bmatrix}1 & 3 & 1\\2 & 0 & 2\end{bmatrix} + \begin{bmatrix}1 & -2 & 3\\1 & 0 & 2\end{bmatrix} = \begin{bmatrix}2 & 1 & 4\\3 & 0 & 4\end{bmatrix}$

A.B does not exist. $A.B^{`} = \begin{bmatrix}1 & 3 & 1\\2 & 0 & 2\end{bmatrix}.\begin{bmatrix}1 & 1\\-2 & 0\\3 & 2\end{bmatrix} = \begin{bmatrix}-2 & 3\\8 & 6\end{bmatrix}$

-- ***(5 Marks)***

Question 2

(a) $\frac{1}{\sqrt{x+9}} = \frac{1}{3}(1+\frac{x}{9})^{-\frac{1}{2}} = \frac{1}{3}\left(1 - \frac{1}{18}x + \frac{1}{4.27}x^2 + \cdots\right), \quad |\frac{x}{9}| < 1$

-- ***(4 Marks)***

(b) $G = \begin{bmatrix} 1 & 2 & -1 & \vdots & 6 \\ 3 & 1 & -1 & \vdots & 2 \\ 2 & -1 & 0 & \vdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 & \vdots & 6 \\ 0 & -5 & 2 & \vdots & -16 \\ 0 & -5 & 2 & \vdots & -11 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 & \vdots & 6 \\ 0 & -5 & 2 & \vdots & -16 \\ 0 & 0 & 0 & \vdots & 4 \end{bmatrix}$

We see that rank A is 2 but rank G is 3. Then, there is no solution.

-- ***(5 Marks)***

(c)(i)Since $u_r = (\mathrm{r}+1)(2+\mathrm{r}^2) = \mathrm{r}^3 + \mathrm{r}^2 + 2\mathrm{r} + 2$

Then $S_n = \frac{1}{4}\mathrm{n}^2(\mathrm{n}+1)^2 + \frac{1}{6}\mathrm{n}(\mathrm{n}+1)(2\mathrm{n}+1) + 2.\frac{1}{2}\mathrm{n}(\mathrm{n}+1) + 2\mathrm{n}$

Then $S = \infty$

$$S_{20} = (100)(21)^2 + \frac{1}{6}(20)(21)(41) + (20)(21) + 40 = 47430$$

-- ***(4 Marks)***

(c)(ii)Since $u_r = \frac{3}{\mathrm{r}^2+\mathrm{r}} = \frac{3}{\mathrm{r}} - \frac{3}{\mathrm{r}+1}$. Then $S_n = \frac{3}{1} - \frac{3}{\mathrm{n}+1}$.

Then $S = 3$ and $S_{20} = 3 - \frac{3}{21} = \frac{60}{21}$

-- ***(4 Marks)***

(d)(i)Prove that: $1 + 2 + 3 + \cdots + \mathrm{n} = \frac{1}{2}\mathrm{n}(\mathrm{n}+1)$

(1)At n = 1, the left hand side (L.H.S) of this relation is 1 and the right hand side (R.H.S) of this relation is 1. Then this relation is true at n = 1.

(2)Assume that this relation is true at n = k.

This means that $1 + 2 + 3 + \cdots + \mathrm{k} = \frac{1}{2}\mathrm{k}(\mathrm{k}+1)$

(3)We shall prove that this relation is true at n = k + 1.

Or prove that, $1 + 2 + 3 + \cdots + (\mathrm{k}+1) = \frac{1}{2}(\mathrm{k}+1)(\mathrm{k}+2)$

From step 2, add k + 1 to both sides, we get

$$1 + 2 + 3 + \cdots + \mathrm{k} + (\mathrm{k}+1) = \frac{1}{2}\mathrm{k}(\mathrm{k}+1) + (\mathrm{k}+1) = \frac{1}{2}(\mathrm{k}+)(\mathrm{k}+2)$$

Then this relation is true for all n.

-- ***(4 Marks)*** (d)(ii) $n^3 + 2n$ is divisible by 3

We want to prove that $\frac{n^3+2n}{3}$ is integer number.

(1)At n = 1, $\frac{1+2}{3} = 1$ which is integer number. Then this relation is true at n = 1.

(2)Assume that this relation is true at n = k.

This means that $\frac{k^3+2k}{3}$ integer number.

(3)We shall prove that this relation is true at n = k + 1.

Or prove that, $\frac{(k+1)^3+2(k+1)}{3}$ is integer number.

Since $(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2$

$$= k^3 + 2k + 3(k^2 + k + 1)$$

Then $\frac{(k+1)^3+2(k+1)}{3} = \frac{k^3+2k+3(k^2+k+1)}{3} = \frac{k^3+2k^2}{3} + k^2 + k + 1$

= integer + integer   From step 2

Then this relation is true for all n.

--- ***(4 Marks)***

Dr. Mohamed Eid

Question 3

(a)Transform the equation $x^2 - 6xy + 9y^2 + 4x + 8y + 15 = 0$ to new axes through $(-2,1)$

Answer

Put $x = x - 2$ and $y = y + 1$

$$(x-2)^2 - 6(x-2)(y+1) + 9(y+1)^2 + 4(x-2) + 8(y+1) + 15 = 0$$

$$\boxed{x^2 - 6xy + 9y^2 - 6x + 38y + 57 = 0}$$

(b)A point moves so that its distance from the $x-$axis is half of its distance from the point $(2,3)$. Find the equation of its locus.

Answer

Let the point is $P(x,y)$ its distance from the $x-$axis is y and its distance from the point

$(2,3)$ is $\sqrt{(x-2)^2+(y-3)^2}$ the locus described by

$$y=\frac{1}{2}\sqrt{(x-2)^2+(y-3)^2}$$

$$4y^2=x^2-4x+4+y^2-6x+9$$

$$\boxed{3y^2-x^2+4x+6y-13=0}$$

(c)Find the value of λ such that the equation $2x^2-3xy-2y^2-8x+6y+\lambda=0$ represent a pair of lines then fined the angle between them.

Answer

$$2x^2-3xy-2y^2-8x+6y+\lambda=(2x+y+c_1)(x-2y+c_2)=0$$

Compare the coefficients in both sides

$c_1+2c_2=-8,\ -2c_1+c_2=6,\quad c_1c_2=\lambda$

Then $\quad c_1=-4,\ c_2=-2,\quad c_1c_2=\lambda=8$

We note that $a+b=0$ the angle between the lines is $\theta=\pi/2$

Another solution

$$2x^2-3xy-2y^2-8x+6y+\lambda=0$$

$a=2, h=-3/2, b=-2, g=-4, f=3$ and $c=\lambda$

Construct the discriminate

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=\begin{vmatrix} 2 & -3/2 & -4 \\ -3/2 & -2 & 3 \\ -4 & 3 & \lambda \end{vmatrix}=0$$

Which show that $\lambda=8$

We note that $a+b=0$ the angle between the lines is $\theta=\pi/2$

(d)Find the equation of the ellipse whose foci $(\pm4,0)$ and its eccentricity is $(1/3)$.

Answer

$(\pm 4,0)=(\pm ae,0)$ then $ae=4$, since $e=(1/3)$ $a=12$

$b^2=a^2(1-e^2)=144(1-\frac{1}{9})=144(\frac{8}{9})=16(8)=128$

The equation is

$$\boxed{\frac{x^2}{144}+\frac{y^2}{128}=1}$$

Question (4)

(a)Find the equation of the tangent to the circle $x^2+y^2=13$ at the point $(2,3)$.

Answer

Equation of the tangent is $xx_1+yy_1=13$ substitute by $(x_1,y_1)=(2,3)$ we get 2x+3y=13

(b)Find the equation of the parabola whose Focusis $F(1,2)$ and directrix is $x+y-2=0$.

Answer

Let the point $P(x,y)$ on the curve then

$$(x-1)^2+(y-2)^2=\left(\frac{x+y-2}{\sqrt{1+1}}\right)^2$$

$$2(x^2+y^2-2x-4y+5)=x^2+y^2+4+2xy-4x-4y$$

$$\boxed{x^2-2xy+y^2+2y+6=0}$$

(c)Find the eccentricities, foci coordinates, directrixfor theellipses $4x^2+9y^2=144$.

Answer

$4x^2+9y^2=144$

Divide the equation by 144

$$\frac{x^2}{36}+\frac{y^2}{16}=1$$

$a^2=36,\ b^2=16$

The center at (0, 0) and the vertices at (±6.0),(0, ±4)

Major axis =2a=12 and minor axis =2b=8

$b^2 = a^2(1-e^2)$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{16}{36}} = \sqrt{\frac{20}{36}} = \frac{\sqrt{5}}{3}$$

Foci at $(\pm ae, 0) = (\pm 2\sqrt{5}, 0)$

Equation of the directrix is $x = \pm\frac{a}{e} = \pm\frac{3\times 6}{\sqrt{5}} = \pm\frac{18}{5}\sqrt{5}$

(d)Find the equation of the tangent to hyperbola $4x^2 - 9y^2 = 36$ which parallel to the line $y = 2x + 3$.

Answer

$a^2 = 9,\ b^2 = 4$

The m equation for the tangent is

$y = mx \pm \sqrt{m^2a^2 - b^2}$

$y = 2x \pm \sqrt{9\times 2 - 4}$

$$\boxed{y = 2x \pm \sqrt{22}}$$

Dr. Fathi Abdsallam